Revue Paralia, Volume 8 (2015) pp s02.1-s02.8
Keywords: Suspended sediment, Vertical profile of the concentration, Non-uniform flow, Transitory state, Deposition, Erosion, Mud, Sand, Model α-β.

Transitory distribution of the suspended sediment in a unidirectional non-uniform flow

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Abstract:
A 1DH modeling is implemented to study a particular case of transport in suspension along a channel corresponding to a 2DV problem. In order to include the vertical dimension, a model named α-β is used. This latter was developed to describe the vertical distribution of suspended sediment in a flow corresponding to an unsteady and/or non-uniform state, and including deposition and resuspension phenomena (SANCHEZ, 2013). Results show that in some regions of the studied domain, the α-β model describing the vertical distribution of suspended sediment, can be simplified because its two parameters (α and β) remain constant.

Translated version of a paper in French (DOI reference: http://dx.doi.org/10.5150/cmcm.2015.016), presented during the edition 3 of the Coastal and Maritime Mediterranean Conference, (25-27 November 2015) in Ferrara, Italy.
Received 02 October 2015, accepted 11 December 2015, available online 30 January 2016.

How to cite this paper:
SANCHEZ M. (2015). *Transitory distribution of the suspended sediment in a unidirectional non-uniform flow*. Revue Paralia, Vol. 8, pp s02.1–s02.8.
DOI: http://dx.doi.org/10.5150/revue-paralia.2015.s02

1. Introduction

Current 2DH hydrodynamic models allow applications on very long periods with fine geometric scale meshes. For these reasons, practical applications on real problems make extensive use of 2DH models. If the variables of the problem are properly integrated onto the vertical coordinate, the results of these models are close to those from the best 3D models. In several types of problem, the vertical dimension can be considered in a realistic and precise manner by this way. Here are two examples:

a) In the wave propagation models describing the vertical distribution of the velocity potential by Stokes-Airy theory (BERKHOFF, 1972).
b) In hydrodynamic flow models that integrate onto the full depth the effect of shear stress on the mean flow velocities (SAINT-VENANT, 1871).

Thanks to the α-β model, which was recently developed (SANCHEZ, 2013), an accurate description of the vertical distribution of sediment in the water column can be obtained. That enables the use of 2DH models for the simulation of sediment transport taking into account (i) aspects linked to a transitory state, (i) convective-diffusive vertical mixing, and (iii) solid exchanges with the bottom (sediment erosion and/or deposition).

The first validations of the α-β model were carried out for unsteady states and low settling velocities W $(0.05<W(\mathrm{mm\ s^{-1}})<3.2)$, which characterizes fine sediments (SANCHEZ, 2013). Subsequently this model was successfully used to simulate the transport in suspension of sand and gravel, under cyclical hydrodynamic actions in a wide range of periods T $(2.5<T(\mathrm{s})<\infty)$, (SANCHEZ, 2014).

The purpose of this article is to study the suspended sediment transport along a channel for non-uniform states. The studied configuration can represent either a river discharging into a lake or an estuary connected with a microtidal sea.

2. Theory for an equilibrium state

If the sedimentary state is uniform and steady, the vertical distribution of the suspended sediment concentration C is governed by:

$$W\,C = -K_z\,\frac{\partial C}{\partial z} \qquad (1)$$

where W is the local mean value of the suspended sediment settling velocity, z is the vertical coordinate and K_z is the turbulent diffusion coefficient in the Oz direction. In the following equations it is assumed that the bottom is located at z=0.

In this study, the physical magnitudes W and K_z are considered to be invariants with z. This hypothesis is a simplification of the problem widely used in practical applications. So, for a sedimentary equilibrium state, the expression for the concentration is:

$$\ddot{C}(z^\circ) = C(0)\exp(-\alpha_\infty\, z^\circ) \qquad (2)$$

where $z°=z/d$ is the non dimensional vertical coordinate, d the depth and α_∞ the Peclet number characterizing the vertical convection-diffusion sediment transfers:

$$\alpha_\infty = \frac{W\,d}{K_z} \tag{3}$$

One usual relation to evaluate K_z retained for this study is:

$$K_z = \frac{\kappa}{6} U_c\, d \tag{4}$$

where U_c is the shear velocity and $\kappa \approx 0{,}4$ the universal Karman constant.

3. Presentation of the α-β model (SANCHEZ, 2013)

Numerical tests carried out during the initial step of validation of the α-β model, show that the concentration C is not well represented by equation 2 for marked transitory hydrodynamic states, even when erosion and deposition are not observed. In agreement with the α-β model, for this specific case concerning transitory states, the concentration, which is then denoted $\tilde{C}$, is correctly modeled by the following equation:

$$\tilde{C}(z°) = C(0)\ \exp(-\alpha\, z°) \tag{5}$$

where α is a parameter whose value always trends to α_∞ in accordance with a phenomenological model, which for a unidirectional 1DH flow (direction Ox), is written as:

$$\frac{\partial \alpha}{\partial t} + \overline{V}_x \frac{\partial \alpha}{\partial x} = c_\alpha \frac{U_c}{d}\left(\alpha_\infty - \alpha\right) \tag{6}$$

where $\overline{V}_x$ is the vertical mean velocity of the flow in the Ox direction, and $c_\alpha \approx 0.667$ is the coefficient of the model.

In most cases, the modifications on the vertical concentration profile induced by deposition and erosion (see Figure 1), are more significant that that produced by the transitory state of the hydrodynamic variables. It is shown that in a general case the profile of C is well described by (SANCHEZ, 2013):

$$C(z°) = C_R\ \exp(-\alpha\, z°)\ \exp(-\beta(1-z°)^2) \tag{7}$$

where C_R is a reference concentration and β is a parameter of the model whose value always trends to β_∞ in accordance with the following phenomenological model including a coefficient c_β:

$$\frac{\partial \beta}{\partial t} + \overline{V}_x \frac{\partial \beta}{\partial x} = c_\beta \frac{U_c}{d}\left(\beta_\infty - \beta\right) \tag{8}$$

where β_∞ is the instantaneous terminal value of the parameter β, which depends on the solid exchanges between the bed and the water column. These exchanges are parameterized by either the deposition rate D^{ef} (sediment sink term in kg m^{-2} s^{-1}) or the erosion rate E^{ef} (sediment source term in kg m^{-2} s^{-1}).

In what follows, the solid exchanges at the bottom are parameterized by a unique exchange rate S^{ef}, which is equal to D^{ef} in case of deposition and to $-E^{ef}$ in case of erosion. Moreover, a law by KRONE (1986) is generalized in order to define a non-dimensional rate for the solid exchanges with the bed:

$$p = \frac{S^{ef}}{W \times C(0)} \qquad (9)$$

It is shown (SANCHEZ, 2013) that α converges to α_∞ and β to β_∞ when the independent variables of the problem (U_c, $\overline{V}_x$, W and d) remain constant on time, provided that parameter p is also constant. In this case, border conditions linked to the solid exchanges at the bottom are exactly satisfied if:

$$\beta_\infty = 0.50\, p\, \alpha \qquad (10)$$

Complementary, in case of erosion or without exchanges between the bed and the water column ($p \leq 0$):

$$c_\beta \approx 0.667 \qquad (11)$$

Finally, in case of deposition ($p>0$):

$$c_\beta \approx 0.667 + 0.3 \times \beta_\infty \qquad (12)$$

4. Methods

A version 1DH of the α-β model is applied to study the suspended sediment transport in a channel by a unidirectional 2DV flow. The imposed configuration for the channel is shown at the top of Figure 2. This channel comprises an erosion region between A and B, a deposition region downstream C, and between these two regions (between B and C) non erosion or deposition is considered. In the simulations, depth d=2 m and settling velocity W=0.001 m s^{-1}, are kept constant. In addition, the hydrodynamic transitional phenomena near the discontinuities, are neglected. Table 1 summarizes the main parameters of the studied problem.

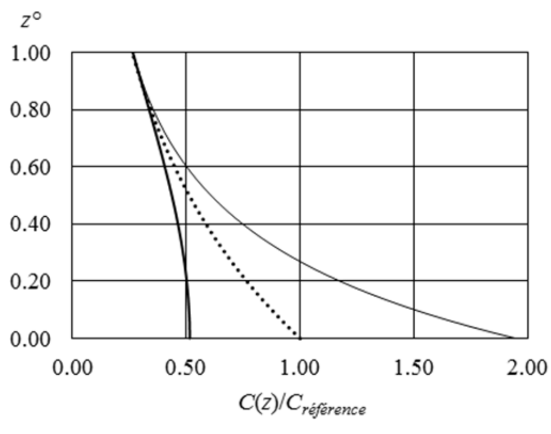

Figure 1. Illustration of the effects of solid exchanges at the bottom on the vertical profile of the suspended sediment concentration. Dotted line: steady state profile. Thick line: profile observed during a deposition period. Thin line: profile observed during an erosion period (z°=0=bottom; z°=1=surface).

Table 1. Summary of hydrosedimentary parameters specific to the studied problem.

	Upstream A	Between A & B	Between B & C	Between C & D	Downstream D
Flow velocity $\overline{V}_x$ ($m\ s^{-1}$)	1.28	1.28	0.64	0.32	0.32
Shear velocity U_c ($m\ s^{-1}$)	0.08	0.08	0.04	0.02	0.02
Length (m)	undefined	760	760	200	undefined
Solid exchange rate, p	0	–1	0	+1	+1

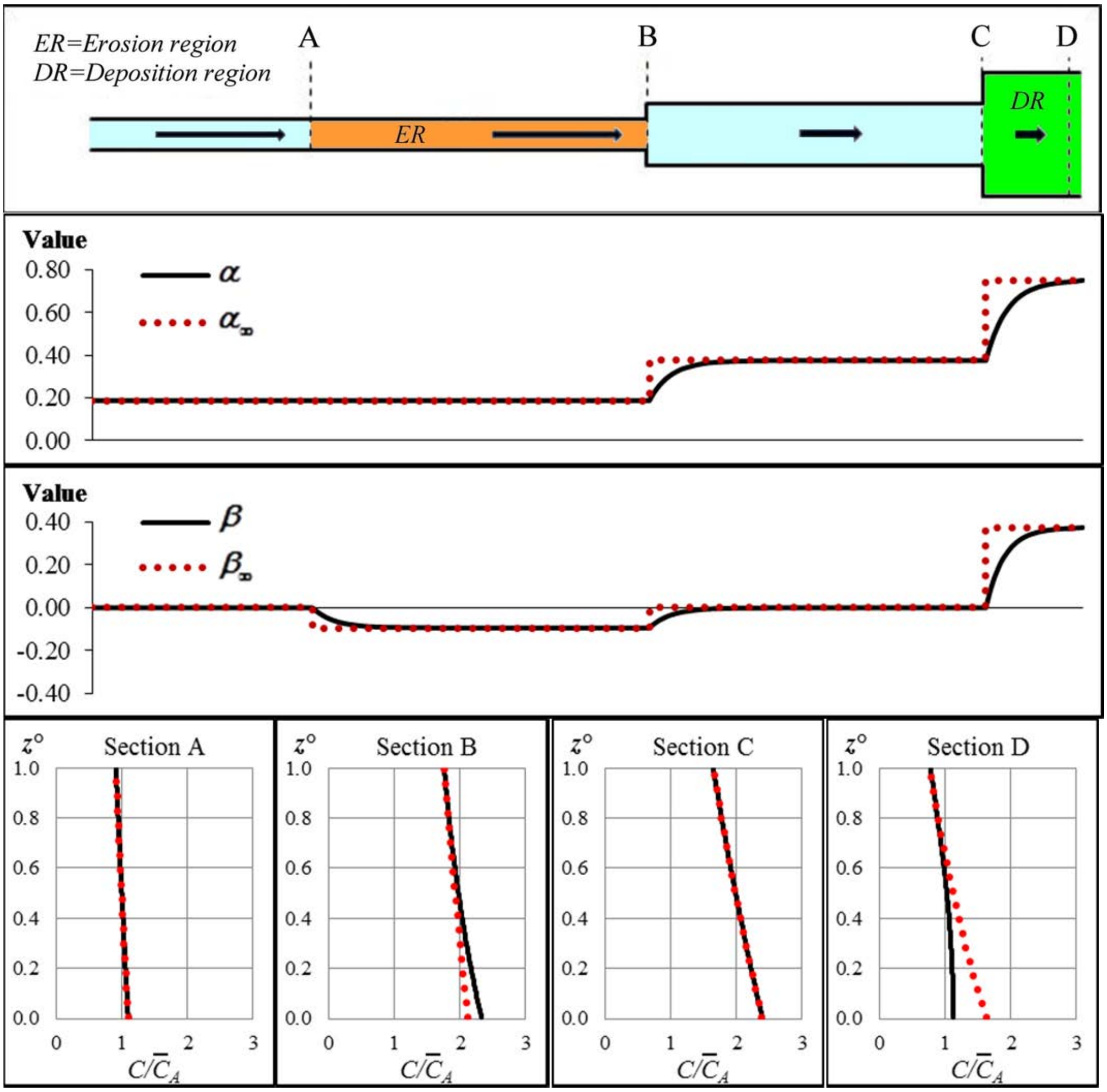

Figure 2. Presentation of results obtained from α-β model.
Top: Sketch of the flow studied along a channel.
2nd row: Evolution of α and α_∞, parameters linked to the convective-diffusive mixing.
3rd row: Evolution of β and β_∞, parameters related to exchanges with the bed.
Bottom: Concentrations reduced by the mean vertical concentration in A as a function of z°, which is shown in the Z-axis; results of α-β model are drawn in black line and the exponential distribution with alpha as the only parameter is plotted as a red dotted line.

5. Discussion of the obtained results

Results obtained for the vertical distribution of the concentration are shown for the cross-sections A, B, C and D (Figure 2, bottom).

The profile for the section A represents the border condition of the problem with an average value of $C/\overline{C}_A$ equal to 1.00 (where $\overline{C}_A$ is the reference concentration upstream the section A). Results from α-β model (black line) are in agreement with the law defined by equation 2 for an equilibrium state (red dotted line).

Due to the simulated erosion between A and B, the average value in B of $C/\overline{C}_A$ reaches about 2.00. It is observed that the concentration at the bottom according to the α-β model is greater than that predicted by an equilibrium state equation and this is explained by a supply of sediment from the bed. The value of $\partial C/\partial z$ in z=0 is in accordance with the border condition for an erosion parameterized with p=-1.

Between sections B and C a transport in suspension without erosion or deposition is imposed, so that the mean vertical value of $C/\overline{C}_A$ remains constant. It gradually moves from a transitory profile downstream B to an equilibrium profile in C. In the point B shear velocity changes from 0.08 m s^{-1} to 0.04 m s^{-1}, and this causes a redistribution of suspended sediments between B and C by increasing sediment accumulation near the bottom. Downstream C, an unhindered free settling is simulated to produce a progressive reduction of the suspended sediments in the flow direction. The vertical mean value of $C/\overline{C}_A$ again becomes equal to 1.00 in section D, where it is verified that the vertical profile of the concentration according to the α-β model, with $\partial C/\partial z$=0 in z=0, is compatible with the border condition for an unhindered deposition, which corresponds to p=+1.

On row 2, Figure 2 shows the evolution of the parameters α and α_∞, which characterize the convective-diffusive vertical transfers. On the one hand, changes in α_∞ close to the flow discontinuities are associated with a locally non-uniform hydrodynamic state. On the other hand, it is noted that following the flow, the model parameter α always converges to its terminal value α_∞.

According to the phenomenological law for the variation of α (Eq. 6), if the independent variables of the problem (U_c, W and d) remain constant over time and along the flow, the time required to reach a steady state for the vertical distribution of suspended sediments from an arbitrary initial condition can be characterized by a time constant $\tau_\alpha = d/(c_\alpha \times U_c)$, so that after a time t=4.6×τ_α it is observed that: $(\alpha_\infty - \alpha) = (\alpha_\infty - \alpha_{init})/100$, where α_{init} is the initial value of the parameter α (at t=0).

In the case of a non-uniform steady flow as the one studied in this article, the path length Λ_α associated with a time t=4.6×τ_α is:

$$\Lambda_\alpha = \frac{4.6 \times d \times \overline{V}_x}{c_\alpha \times U_c} \tag{13}$$

Finally, on row 3, Figure 2 shows the evolution of β and β_∞, parameters related to sediment exchanges with the bed. It is observed that the model parameter β always converges to its terminal value β_∞. Variations on β_∞ are directly linked to variations in the parameter *p* characterizing these solid exchanges. With erosion or deposition from an arbitrary initial condition characterized by $\beta=\beta_{init}$ at $t=0$, the path length Λ_β required to observe a β value as $(\beta_\infty-\beta)=(\beta_\infty-\beta_{init})/100$, is:

$$\Lambda_\beta = \frac{4.6\times d\times \overline{V}_x}{c_\beta \times U_c} \tag{14}$$

For instance, we can cite a specific case with $c_\alpha=c_\beta=0{,}667$ and $\overline{V}_x/U_c=16$, for which computations give $\Lambda_\alpha=\Lambda_\beta=110\times d$.

6. Conclusion

The numerical simulation carried out with the α-β model corresponding to a steady non-uniform flow, shows that the bottom boundary condition related to solid exchanges (erosion and deposition) can adequately be described using Equation 7.

If the parameter *p* characterizing the solid exchanges with the bed and the independent variables of the problem (U_c, W and d), all vary gradually along the flow, the vertical distribution of suspended sediment is correctly described using the following simplifications: $\alpha\approx\alpha_\infty$ and $\beta\approx\beta_\infty$.

More precisely, if variations $\Delta\alpha_\infty$ of the parameter α_∞ along a length Λ_α following the flow are everywhere such as $\mathrm{Abs}(\Delta\alpha_\infty/\alpha_\infty)<<1$, then everywhere $\alpha\approx\alpha_\infty$.

In the same way, if variations $\Delta\beta_\infty$ of the parameter β_∞ along a length Λ_β following the flow are everywhere such as $\mathrm{Abs}(\Delta\beta_\infty/\beta_\infty)<<1$, then everywhere $\beta\approx\beta_\infty$.

It should be noted that the expressions giving Λ_α and Λ_β (equations 13 and 14, respectively) are derived from this study.

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