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Nonlinear modelling of interaction of waves with a moving submerged body

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Abstract:
The work presented herein relates to the development of an advanced simulation code allowing a description of the motion of a submerged body under the action of waves, including large oscillations. The long-term goal of this tool is to model the behaviour of certain types of submerged Wave Energy Recovery Systems (WERS).
In this study a potential flow approach was adopted to describe the hydrodynamic part, limited to 2DV (i.e. in the vertical plane), corresponding to the case of a numerical wave tank. The model used to generate and propagate waves is a fully nonlinear potential flow model, based on a high-order boundary element method developed by Grilli and his colleagues over the past 20 years. This model, which has already been largely validated for a number of different oceanic and coastal applications, has been modified to take into account the presence of either a fixed or moving rigid submerged body, by including the computation of the hydrodynamic forces acting on the body. A specific methodology has been developed to solve for the coupled hydrodynamic-mechanical problem.
Two validation cases are presented and results of the numerical model are compared to those of other mathematical models: (i) the analytical theory by WU (1993), for a cylinder in a prescribed circular motion; and (ii) the linear theory by EVANS et al. (1979), for the case of a cylinder submitted to external forces (spring-like force and forces related to wave-energy extraction, …) in addition to hydrodynamic forces.

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1. Introduction

Over the past few years, there has been a marked resurgence of interest in marine renewable energy, in particular wave energy, with the development and testing of a number of different WECs. Amongst the different technologies proposed, are the "point absorber" type WECs that use the oscillating motion of floating or submerged bodies under the impact of propagating waves. These bodies are often submerged at minimal depth below the water surface and undergo large amplitude oscillatory motions (for example, the CETO system, MANN *et al*., 2007). As a result, models based on the hypothesis of small amplitude motion and/or a linearised free surface are not adapted to the simulation of the coupled hydrodynamic-body dynamic system. The work presented herein consists in the development of a numerical model capable of nonlinear wave-body interaction computations in two dimensional space (2DV), which can then be applied for modelling WECs dynamics in real sea states (irregular waves).

A first step in this project consists of solving the coupled nonlinear problem for the case of a submerged horizontal cylinder. The principal characteristics of the numerical model developed are presented hereafter, as well as the two validation cases for a submerged cylinder of circular section, the first case being for prescribed motion and the second for "free" motion under the effect of several forces.

The calculation of nonlinear interactions between waves and a cylindrical body has been a subject of interest for numerous authors. For instance, CHAPLIN (1984) carried out a series of tests in a wave tank, specifically measuring the influence of the Keulegan-Carpenter (K_C) number on the hydrodynamic forces exercised on the body. More recently, WU (1993) adopted an analytic approach to calculate the hydrodynamic forces exercised on a cylinder undergoing large-amplitude motions, using linearised free surface conditions and body boundary conditions satisfied on its instantaneous position. The work of these two authors has been widely used to validate a number of numerical models, such as the Sindbad model (COINTE, 1989) based on the Boundary Element Method (BEM), or again the model developed by KENT & CHOI (2007) employing a High-Order Spectral method (HOS).

2. Mathematical formulation of the coupled problem

2.1 Outline of the model

We make use of a *Numerical Wave Tank* (NWT) based on a *Fully Nonlinear Potential Flow* (FNPF) model, applied to the computation of nonlinear interactions (hydrodynamic forces and induced motion) between the wave-induced flow and a submerged body representing a vertical cross-section of a WEC moored on the sea bottom. This is an extension of the model developed over the last 20 years by Grilli and co-workers (GRILLI & SUBRAMANYA, 1996; GRILLI, 1997; GRILLI & HORRILLO, 1997). The equations of the model are briefly presented hereafter (for additional details, please refer to the above mentioned references). The velocity potential $\phi(\underline{x},t)$ is used to represent a flow, which is assumed to be irrotational and incompressible, in the vertical plane (x, z), and the velocity field is denoted as $\underline{u} = \nabla\phi = (u, w)$.

The continuity equation over the closed fluid domain $\Omega(t)$, with boundaries $\Gamma(t)$, reads as a Laplace equation for the potential (see Fig. 1 for a definition sketch of the computational domain):

$$\nabla^2\phi = 0 \qquad \text{on } \Omega(t) \tag{1}$$

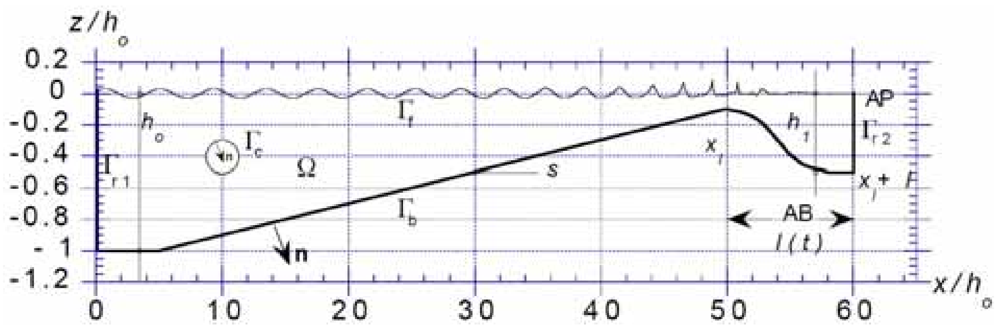

Figure 1. Sketch of the computational domain $\Omega(t)$ and associated boundaries.

On the (time varying) free surface $\Gamma_f(t)$, the potential satisfies the kinematic and dynamic free surface boundary conditions:

$$\begin{cases} \dfrac{D\underline{r}}{Dt} = (\dfrac{\partial}{\partial t} + \underline{u}\nabla)\underline{r} = \underline{u} = \nabla\phi \\ \dfrac{D\phi}{Dt} = -gz + \dfrac{1}{2}\nabla\phi\cdot\nabla\phi + \dfrac{p_a}{\rho} \end{cases} \qquad \text{on } \Gamma_f(t) \tag{2}$$

respectively, with $\underline{r}$ being the position vector of a point on the free surface, g the acceleration of gravity, p_a the atmospheric pressure and ρ the fluid density. At the bottom, assumed to be steady, a slip condition is imposed:

$$\frac{\partial\phi}{\partial n} = \nabla\phi\cdot\underline{n} = 0 \qquad \text{on } \Gamma_b \tag{3}$$

$\underline{n}$ stands for the normal unit vector at the boundary, pointing outwards of the fluid domain.

On the left lateral boundary of the domain $\Gamma_{r1}(t)$, periodic or irregular waves can be generated through the motion of a flap-type or piston-type plane wavemaker. It is also possible to use an exact computational method for wave profile and kinematics based on the Stream Function approach. At the right lateral boundary of the domain, an absorbing beach (*AB*) is implemented in order to reduce the reflection of waves on the boundary $\Gamma_{r2}(t)$. More information on the generation and absorption of waves in the model are available in GRILLI & HORRILLO (1997). On the boundary of the submerged body $\Gamma_c(t)$, a specific condition is imposed, which is described in the next section.

Equation (1) is transformed into a Boundary Integral Equation (BIE) by applying Green's second identity; it is further solved by the Boundary Element Method (BEM). The BIE is thus evaluated at N nodes on the boundary and M higher-order elements are defined to interpolate in between discretization nodes. In the following, quadratic isoparametric elements are used on lateral and bottom boundaries. Isoparametric linear elements are used on the body boundary and cubic elements on the free surface in order to ensure continuity of the boundary slope. In these latter elements, referred to as Mixed Cubic Interpolation (MCI) elements, geometry is modelled by cubic splines and field variables are interpolated between each pair of nodes, using the mid-section of a four-node "sliding" isoparametric element. Expressions of the various integrals over the elements (regular, singular and quasi-singular) are given in GRILLI & SUBRAMANYA (1996).

The free surface boundary conditions (2) are marched in time by using second-order Taylor series, in which appear the time step Δt and the Lagrangian time derivatives of ϕ and $\underline{r}$. Second order terms are obtained from the Lagrangian time differentiation of equations (2), and computed by solving a second BIE on $(\partial\phi/\partial t, \partial^2\phi/\partial t\partial n)$, whose boundary conditions are formulated by using the results from the first BIE. Detailed expressions of these Taylor series are given in GRILLI (1997).

This numerical wave tank has been modified to include a rigid body, fully submerged under the free surface. Two situations are considered: (i) the case of a body in forced (prescribed) motion (which includes the case of a fixed body); and (ii) the case of a "freely" moving body (under the actions of the various forces exerted on it).

2.2 Body in prescribed motion

When the motion of the body is prescribed, a Neumann-type condition is imposed on the body boundary for the normal flux of the potential:

$$\frac{\partial\phi}{\partial n} = \underline{\dot{\alpha}} \cdot \underline{n} \qquad \text{on } \Gamma_c(t) \tag{4}$$

where $\underline{\dot{\alpha}}$ stands for the velocity vector of a node on the body boundary, which is known in this case of forced motion. As a second Laplace problem is solved for $(\partial\phi/\partial t, \partial^2\phi/\partial t\partial n)$, an additional condition has to be specified to compute $\partial\phi/\partial t$. Following COINTE (1989), VAN DAALEN (1993) and TANIZAWA (1995), a Neumann-type condition on $\partial^2\phi/\partial t\partial n$ is imposed on the body boundary:

$$\frac{\partial^2\phi}{\partial t\partial n} = \underline{\ddot{\alpha}} \cdot \underline{n} + \dot{\theta}\left(\underline{\dot{\alpha}} \cdot \underline{s} - \frac{\partial\phi}{\partial s}\right) - \left(\frac{1}{R}\frac{\partial\phi}{\partial s} + \frac{\partial^2\phi}{\partial s\partial n}\right)\underline{\dot{\alpha}} \cdot \underline{s} + \left(\frac{\partial^2\phi}{\partial s^2} - \frac{1}{R}\frac{\partial\phi}{\partial n}\right)\underline{\dot{\alpha}} \cdot \underline{n} \quad \text{on } \Gamma_c(t) \tag{5}$$

where $\underline{\ddot{\alpha}}$ and $\dot{\theta}$ are the solid acceleration of the node on the body boundary and the rotation velocity of the body, respectively; $1/R$ is the local curvature of the body boundary, and $\underline{s}$ and $\underline{n}$ are the unit vectors, respectively, normal and tangent at the boundary $\Gamma_c(t)$. Equations (4) and (5) are similar to the boundary conditions used for the generation of waves by a plane wavemaker (GRILLI & HORRILLO, 1997; GRILLI, 1997).

2.3 Body in "free" motion

When no motion is imposed to the body, the problem is referred to as a *freely-moving body* problem. In this case, the acceleration $\underline{\ddot{\alpha}}$ is unknown, and equation (5) cannot directly be used as a boundary condition to solve the Laplace problem for $\partial\phi/\partial t$. Furthermore, the velocity $\underline{\dot{\alpha}}$ in equation (4) needs to be solved as part of a coupled fluid-structure problem.

Considering a rigid body of mass M and mass moment of inertia I about its centre of mass, the dynamic equations governing the body motion read:

$$\begin{cases} M\,\underline{\ddot{x}} = \oint_{\Gamma_c} p\underline{n}d\Gamma_c + M\,\underline{g} + \underline{F}_{ext} \\ I\ddot{\theta}\underline{e}_y = -\oint_{\Gamma_c} p(\underline{r}\times\underline{n})d\Gamma_c + \underline{M}_{ext} \end{cases} \tag{6}$$

where $\underline{\ddot{x}}$ is the acceleration of the body centre of mass, $\ddot{\theta}$ is the angular acceleration of the solid body about the centre of mass, $\underline{F}_{ext}$ and $\underline{M}_{\mathrm{ext}}$ represent any kind of applied external force and momentum acting on the body (e.g. mooring, force modelling power take-off, etc.), $\underline{r}$ is the position vector of a point on the body boundary with respect to the centre of mass, and $\underline{e}_y$ is the unit vector normal to the plane (x, z), and defined by $\underline{e}_y = \underline{e}_z \times \underline{e}_x$. Finally, the pressure p along the body boundary is given by the (nonlinear) Bernoulli equation:

$$p = -\rho\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\cdot\nabla\phi + gz\right) \tag{7}$$

The computation of the pressure is rendered difficult, due to the fact that $\partial\phi/\partial t$ is unknown at any given time along $\Gamma_c(t)$. Furthermore, in the second Laplace problem on $\partial\phi/\partial t$, the Neumann condition (5) on the body boundary is unknown as well. Several authors have proposed methods to overcome this difficulty: (i) the mode decomposition method (VINJE & BREVIG, 1981); (ii) the iterative method (SEN, 1993; CAO *et al.*, 1994); (iii) the indirect method (WU & EATOCK-TAYLOR, 1996); and (iv) the implicit method (VAN DAALEN, 1993; TANIZAWA, 1995). The latter has been chosen here, as no iterations are required and there is no need to introduce any artificial potential. This method combines equations (5), (6) and (7) to derive a new Boundary Integral Equation of the following form:

$$\frac{\partial^2\phi}{\partial t\partial n}(\underline{x}) + \oint_{\Gamma_c} K(\underline{x},\underline{\xi})\frac{\partial\phi}{\partial t}d\Gamma_c(\underline{\xi}) = \gamma(\underline{x}) \tag{8}$$

This equation is then discretized on the body boundary $\Gamma_c(t)$ using the BEM elements. The matrix K is regular, symmetric, and depends only upon the shape of the body. The function γ is explicitly evaluated at each iteration. After solving the first Laplace problem for the potential, equation (8) is added to the matrix equation of the problem for $\partial\phi/\partial t$ so as to have as many equations as unknowns. This second linear system is solved with a LU decomposition scheme based on the direct elimination technique proposed by Khaletski. After having computed the pressure using equation (7), equations of body motion (6) are marched in time and provide the position and velocity of the body at the next time step. Various schemes have been compared on simple cases of mechanical oscillators with damping, and the Newmark scheme was retained:

$$\begin{cases} \underline{\dot{x}}_{n+1} = \underline{\dot{x}}_n + \Delta t\left((1-\gamma)\underline{\ddot{x}}_n + \gamma\underline{\ddot{x}}_{n+1}\right) \\ \underline{x}_{n+1} = \underline{x}_n + \Delta t\underline{\dot{x}}_n + \Delta t^2\left((\frac{1}{2}-\beta)\underline{\ddot{x}}_n + \beta\underline{\ddot{x}}_{n+1}\right) \end{cases} \tag{9}$$

Parameters were selected as $\gamma = 1/2$ and $\beta = 1/4$, corresponding to the so-called mean acceleration method, which makes this scheme second order in time and unconditionally stable, at least for linear dynamical systems. As this scheme is implicit ($\underline{\ddot{x}}_{n+1}$ is unknown at the beginning of the time step), iterations are required. As initial values, we use time polynomial extrapolations based on the values of the unknown quantities at the fifth previous time steps. With this choice, few iterations are required to achieve convergence (*i.e.* 1 to 3 iterations, in these simulations).

The time step actually used is fixed by the hydrodynamic solver, and updated at each iteration as a function of an optimal Courant number C_0 (about 0.45) and the minimal distance between two adjacent nodes on the free surface $\Delta r_{\min}$:

$$\Delta t = C_0 \frac{\Delta r_{\min}}{\sqrt{gd}} \tag{10}$$

where d is the local water depth. More information on the stability and convergence of the temporal scheme used by the hydrodynamic solver is available in GRILLI (1997).

3. Case of a cylinder in prescribed motion

The radiation problem resulting from large amplitude motions of a cylinder submerged in a fluid of infinite depth, under a free surface initially at rest, was treated analytically by WU (1993). No approximations are made for the body boundary conditions; however, Wu uses linearised free surface conditions. He formulates the solution for the potential flow by way of an expansion into generalised spherical multi-poles and calculates the hydrodynamic forces exercised on a cylinder of radius R undergoing circular rotation in a clockwise direction and with an orbit of radius C. Wu takes into account a single wave number value kR=0.5 and 8 non-dimensional orbits C/R. The wave number of the waves generated by the cylinder is linked to the frequency of the circular motion by the infinite depth linear dispersion relation $k=\omega^2/g$. Wu demonstrated that, starting from a free surface at rest, waves are generated only towards the right of the flume; this was also confirmed by the simulations performed for this study (Fig. 2).

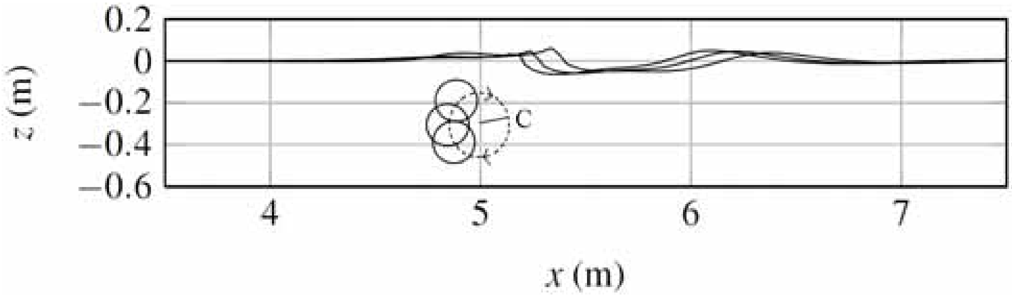

Figure 2. Position of the cylinder and the free surface at three successive time instants (t/T=4.40 ; t/T=4.49 ; t/T=4.62).

These simulations were performed in a wave canal of length L=20 m and depth d=3 m, with an absorbing beach installed over the last 7 metres. A cylinder of radius R=0.1 m is placed 5 m from the left boundary and at z_c=-3R beneath the free surface at rest. The cylinder is then progressively put into motion over four rotation periods (in order to avoid instabilities linked to an abrupt start-up), to achieve a final circular orbit with an angular velocity ω. The infinite depth hypothesis proposed by Wu is clearly verified in these simulations with kd=5. In the BEM, 200 nodes over the free surface and 80 nodes over the body were used. The simulations covered 15 rotation periods.
Similar to Wu's analysis, the non-dimensional vertical and horizontal forces were decomposed in Fourier series, in the form:

$$\begin{cases} \dfrac{F_x}{C\rho\pi R^2\omega^2} = F_x^{(0)} + \sum\limits_{n\geq 1} F_x^{(n)} \cos(n\omega t + \varphi_n) \\ \dfrac{F_z}{C\rho\pi R^2\omega^2} = F_z^{(0)} + \sum\limits_{n\geq 1} F_z^{(n)} \sin(n\omega t + \varphi_n) \end{cases} \tag{11}$$

Figures 3a, 3b and 3c compare the mean values as well as the amplitudes of the first two harmonics of the non-dimensional force components to the WU (1993) results for the 8 different motion amplitudes.
These results show good agreement with WU's (1993) theoretical results in the case of relatively small amplitudes (C/R<1). Above these amplitudes, there is a significant movement of the cylinder towards the free surface and non-linear effects are no longer negligible, as can be seen in figure 4, which shows the simulated vertical and horizontal forces.
The results obtained with the HOS method (KENT & CHOI, 2007) were also compared to Wu's results, unfortunately for orbital radii inferior to C/R=0.6, amplitude at which non-linear effects do not appear clearly.

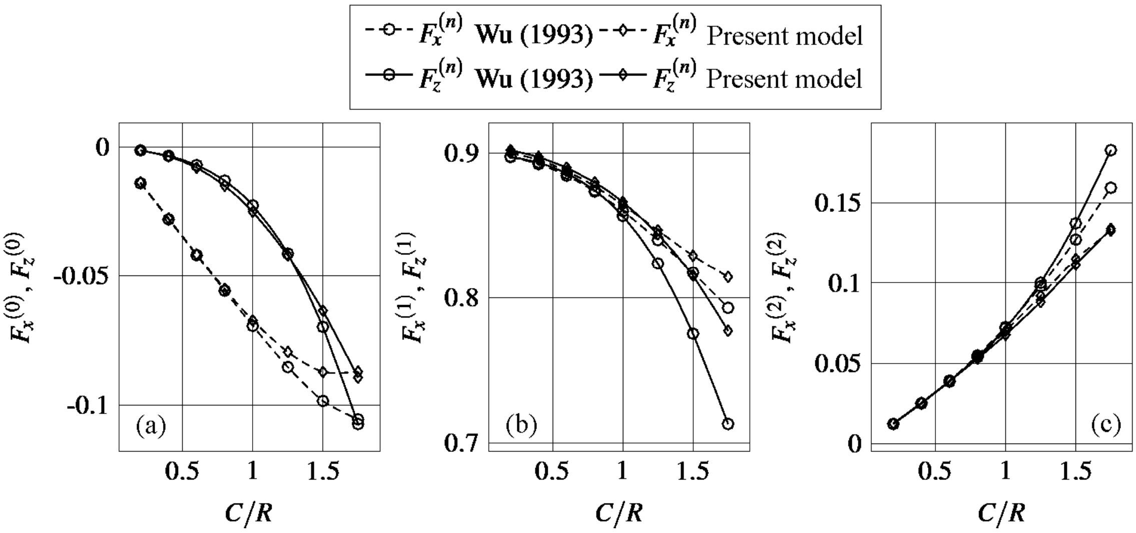

Figure 3. Mean value (a), amplitude of the 1st harmonic (b) and of the 2nd harmonic (c) of horizontal and vertical non-dimensional force components as a function of the non-dimensional orbital radius C/R, compared with results from theory by WU (1993).

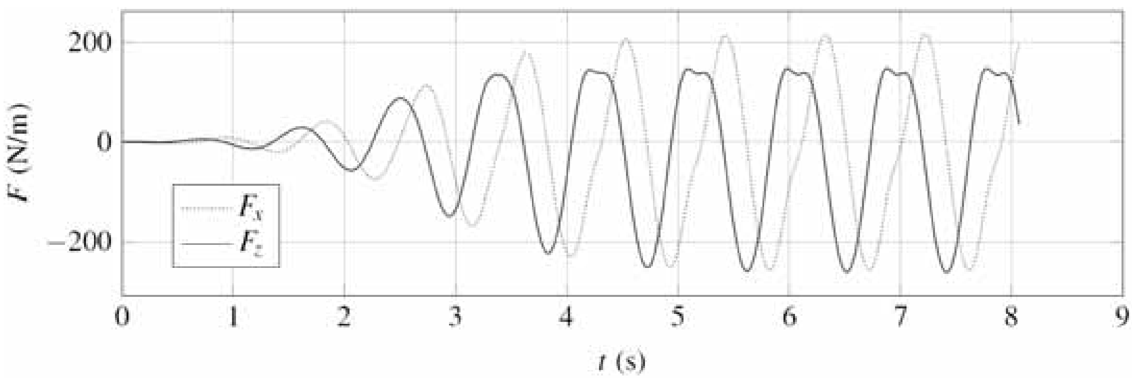

Figure 4. Vertical and horizontal forces, kR=0.5, C/R=1.75.

4. Case of cylinder in free motion

As seen in the previous section, a submerged cylinder undergoing forced circular motion acts as a unidirectional wavemaker. Conversely, it is envisageable that a submerged cylinder placed in a wave field and suitably constrained by anchors on the bottom or connected to a fixed structure using a spring/damper combination system could efficiently absorb wave energy, and could have the potential to act as a WEC. This idea was introduced and studied in the 1980s, and is known as the "Bristol cylinder" (EVANS *et al.*, 1979). The 1st order (linear) solution developed by Evans *et al.* is used as a comparison for the present simulations. Here, the motion of a submerged circular cylinder under regular waves is considered. The equation of motion of such a cylinder, assuming mass M per unit length, at position $\underline{x}$ at time t starting from its initial resting position $\underline{x}_{ini}$, is governed by:

$$M\,\ddot{\underline{x}} = \underline{F}_h + M\,\underline{g} - d_0\,\dot{\underline{x}} - k_0(\underline{x} - \underline{x}_{ini}) \qquad (12)$$

$\underline{F}_h$ represents the hydrodynamic force induced by the waves, and (k_0, d_0) are the stiffness and damping constants, respectively, considered to be identical in the x and z directions, and computed for a given tuning angular frequency $\omega_0 = 2\pi f_0$, for which the power absorbed by the cylinder is maximal (EVANS *et al.*, 1979):

$$\begin{cases} k_0 = \left(M + a_{ii}(\omega_0)\right)\omega_0^{\,2} \\ d_0 = b_{ii}(\omega_0) \end{cases} \tag{13}$$

where $a_{ii}(\omega_0)$ and $b_{ii}(\omega_0)$ are the (linear) added mass and the radiation damping coefficient, respectively of the cylinder, at the tuning frequency. EVANS *et al.* (1979) showed that, under these conditions, the centre of the cylinder moves along a circle of radius *C*, when excited by linear monochromatic waves of amplitude *A* and angular frequency ω:

$$\left(\frac{C}{A}\right)^2 = \frac{\rho g^2 b_{ii}(\omega)}{\omega^3\left\{\left(d_0 + b_{ii}(\omega)\right)^2 + \dfrac{1}{\omega^2}\left[k_0 - \left(M + a_{ii}(\omega)\right)\omega^2\right]^2\right\}} \tag{14}$$

The configuration used by Evans *et al.* is reproduced in the present simulations: a cylinder of radius R=0.05 m, tuned at the frequency f_0=1.65 Hz, is placed at the initial position z_c=–0.0625 m beneath the free surface at rest in a flume of length 20 m and depth 0.60 m, with an absorbing beach specified over the last 7 metres. Monochromatic waves with a very low amplitude (A/R=0.0033) are generated with 8 different frequencies ranging between 1 Hz and 2 Hz.

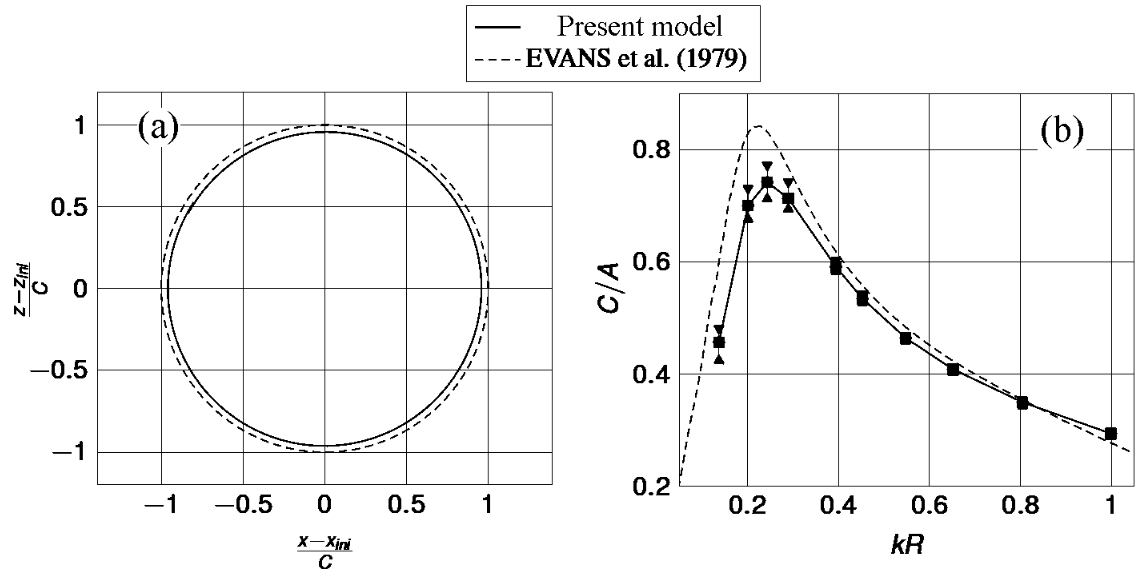

Figure 5. (a) Trajectory of the centre of mass of the cylinder tuned at the frequency f_0=1.65 Hz over a wave period (case f=f_0,, i.e. kR=0.55)
(b) Evolution of the radius of the trajectory for different wave frequencies.
The tuning frequency corresponds to kR=0.55. Symbols refer to minimum, mean and maximum simulated radii over a wave period.

After a time transient period, the centre of the cylinder follows a quasi-circular and stable-in-time path, plotted in figure 5a (case $f=f_0$, *i.e.* $kR=0.55$). This form of trajectory is observed for the 8 considered frequencies. Figure 5b shows the evolution of the radius C of the trajectory (made non-dimensional by the radius R of the cylinder) as a function of kR, compared to the linear theory of EVANS *et al*. (1979). The computed radius is slightly less than the linear theory predicted radius; this is probably related to nonlinear effects (presently under consideration).
This first test-case confirms the ability of the model to simulate the dynamics of a body in free motion responding to wave action (and other forces). It is also the first application of the model to a realistic case, with the long-term goal of modelling the dynamics of submerged WEC systems.

5. Conclusions and outlook of future developments

The numerical model, namely a two-dimensional numerical wave tank based on a fully nonlinear potential flow theory, has been described. The coupled hydro-mechanical problem, related to the presence of a fully submerged cylindrical body either in prescribed or "free" motion, has been mathematically formulated and numerically solved by using the implicit method proposed by VAN DAALEN (1993) and TANIZAWA (1995). The numerical model results were then compared to WU's (1993) theoretical results for the case of a cylinder in prescribed motion, and to the linear theory solution of EVANS *et al.* (1979) for the "free" motion case (with spring and damping in the horizontal and vertical directions), representing a schematic WEC.
Ongoing work is currently devoted to the extension of the model to simulating irregular sea states (defined by a given energy density spectrum), namely the possibility of taking viscous effects into account through an appropriate additional force term, as well as the modelling of more realistic WEC systems. The model will then be extended to enable simulation of 3D problems.

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